Testing a Riemannian twisted solar loop model from EUV data and magnetic topology

by

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Abstract

Compact Riemannian solar twisted magnetic flux tube surfaces model are tested against solar extreme ultraviolet (EUV) lines observations, allowing us to compute the diameter and height of solar plasma loops. The relation between magnetic and torsion energies is found for a nonplanar solar twisted (torsioned) loop to be 10^9, which shows that the contribution of torsion energy to the solar loop is extremely weaker than the magnetic energy contribution. In this case solar loops of up $5000km$ in diameter can be reached. The height of $220.000km$ is used to obtain an estimate for torsion based on the Riemannian flux tube surface, which yields $\tau_0 = 0.9{\times}10^{-8}m^{-1}$ which coincides with one of the data of $(0.9{\pm}0.4){\times}10^{-8}m^{-1}$ obtained by Lopez-Fuentes et al (2003). This result tells us that the Riemannian flux tube model for plasma solar loops is consistent with experimental results in solar physics. These results are obtained for a homogeneous twisted solar loop. By making use of Moffatt-Ricca theorem for the bounds on torsional energy of unknotted vortex filaments, applied to magnetic topology, one places bounds on the lengths of EUV solar loops. New results as the vorticity of the plasma flow along the tube is also computed in terms of the flux tube twist. **PACS numbers:**

Astrophysical plasmas: 95.30.Qd. 02.40.Hw:Riemannian geometries

I Introduction

A simple Riemannian twisted magnetic flux tube model has been recently investigated by Ricca [1, 2] as a model for solar loops and the inflexional desiquilibrium of these loops. The new feature of these models is that they incorporate the twist and its torsion contribution to the tube. Riccas model has recently been theoretically tested [3] as a model for electric currents [4] in solar loops as well as to a dynamo [5] flux tube model which can be obtained by conformal mappings on the Riemannian flux tube, which includes generalized Arnold fast dynamo [6] solution. In this brief report we consider another test for the Riemannian flux tube model, this time against the EUV line observations, where the twist (torsion) of the nonplanar loop is taken from the detailed analysis of Lopez-Fuentes et al [7] based on the fact that the magnetic axis of the tube flow possesses Frenet curvature and torsion is presented. Throughout this report we use the thin tube approximation which is fully justifiable numerically in the paper by EUV lines data [8]. The Moffatt-Ricca theorem [9] on unknowtted vortex fluid filaments applied to magnetohydrodynamic (MHD), where the torsioned magnetic flux tube axis is assumed to be a vortex plasma filament. The lower limit of the loop length is given by the EUV solar loops. From the magnetic topology of solar loops [10] we therefore are able to test magnetic flux tube models. . Throughout the paper we use the force-free equation $\nabla \times \vec{B} = \alpha_{twist} \vec{B}$ where α_{twist} is constant. This equation is used in most of the references in solar physics [7, 8, 10] the torsion of the helical magnetic field. The main mathematical distinction between twist and torsion resides in the fact that the first is a measured of how lines around an axis rotate along this very same axis and therefore is a topological concept that requires at least two lines, while torsion requires just one line to be built. Based on this topological idea we shall proceed deriving equations between torsion and twist and computing the solution. In the case of helical solar dynamos considered here, we show from observational results from TRACE, that torsion is well-within the twist limit obtained by Lopez-Fuentes et al [8]. This report is organised as follows: Section II presents the computation of twist or helicity equation above in the coordinates of the flux tube. Section III deals with the testing of the plasma flux tube against the EUV data and unknotted loops. Section IV presents the conclusions.

II The Riemannian plasma loop metric model

In this section we make a brief review of the Riemannian tube metric In this section we shall consider the twisted flux tube Riemann metric. The metric $g(X,Y)$ line element can be defined as [1, 2]

$$ds^2 = dr^2 + r^2 d{\theta_R}^2 + K^2(s) ds^2 \tag{II.1}$$

This line element was used previously by Ricca [1] and the author [3] as a magnetic flux tubes with applications in solar and plasma astrophysics. This is a Riemannian line element

$$ds^2 = g_{ij} dx^i dx^j \tag{II.2}$$

if the tube coordinates are (r, θ_R, s) [2] where $\theta(s) = \theta_R - \int \tau ds$ where τ is the Frenet torsion of the tube axis and $K(s)$ is given by

$$K^2(s) = [1 - r\kappa(s) cos\theta(s)]^2 \tag{II.3}$$

Here we shall make use of the thin approximation of nonplanar twisted magnetic flux tube. Recently Toeroek and Kliem [11] found that by using TRACE solar satellite 195 angstrom line an unstable kink solar loop was found where twist has a geometrical expression in the satellite images. from an untwisted tube by stationary, very slow, perturbation of the equations of force-free magnetic fields [7]. Let us consider the Lorentz magnetic force given by

$$\vec{F} = [\nabla \times \vec{B}] \times \vec{B} \tag{II.4}$$

which shows clearly that if the magnetic field obeys the law

$$\nabla \times \vec{B} = \alpha_{twist} \vec{B} \tag{II.5}$$

force in equation (II.4) vanishes. Let us now consider the solenoidal equation for $\vec{B}$ which is given by

$$\vec{B} = \vec{e_\theta} B_\theta + \vec{t} B_s \tag{II.6}$$

which obeys

$$\partial_s B_\theta = B_\theta \tau_0 \kappa_0 r_0 sin\theta \tag{II.7}$$

where r_0 is consider half the height of the plasma loop above the sun surface. Now expressing equation (II.5) in terms of the Riemannian metric of the solar loop and splitting it into three scalar MHD equations along the Frenet frame, yields

$$\frac{B_\theta}{B_s} = \alpha_{twist}\tau_0 r_0 \tag{II.8}$$

$$\alpha_{twist} = [-\frac{\tau_0 cos\theta}{r_0} + (\Omega(s) + \tau_0)] \tag{II.9}$$

$$\alpha_{twist} cos\theta = [\tau_0 \frac{B_\theta}{B_s} + \frac{sin\theta}{r_0}] \tag{II.10}$$

where $\Omega(s)$ represents the vorticity of the plasma flow about the magnetic flux tube axis. Algebraic manipulation of those last two equations yields the following result

$$\alpha_{twist} + \frac{cos\theta}{r_0} = \Omega(s) \tag{II.11}$$

This shows clearly that the vorticity of the plasma flow inside the loop depends upon the twist of the solar loops. In the next section we apply these theoretical results to solar data to test the model.

III Testing the Riemannian plasma loop model

In this section we shall consider that our plasma loop is a solar EUV loop and shall take advantage of some known data to show that twist coincides with the Frenet torsion in modulus , we also see from the last section that the vorticity contains a small contribution of the twist since the co-sine term dominates in equation (II.11). Since the flux tube is helicoidal [12] one is able to use the identity

$$\tau_0 = \kappa_0 = \frac{1}{R} \tag{III.12}$$

Thus thanks to EUV solar loops data one is able to compute the torsion of the EUV solar loop from the height of the solar loop which is about $220,000km$ which upon substitution into (III.12) yields a torsion value of the order $\tau_0 = 0.9{\times}10^{-8}m^{-1}$ which well within the twist result obtained by Lopez-Fuentes et al [8] with minus sign which is $-0.9 \pm 0.4{\times}10^{-1}m^{-1}$. This data was obtained on the active region $AR7790$ on 18/10/94. Another

interesting consequency of the Riemannian solar loop model is that from dynamo action expression (II.8) one is able to obtain the dynamo relation

$$\frac{B_{\theta}}{B_{s}} = {\alpha^{2}}_{twist} r_{0} = 0.81{\times}10^{-10} \tag{III.13}$$

This means that the poloidal component of the magnetic field is extremely weaker than the toroidal field B_s. Other interesting test of plasma loop Riemannian model is given by the Moffatt and Ricca theorem for unknotted filaments which states that

$$\int \tau_{0} ds \geq 2\pi \tag{III.14}$$

The integral on the LHS of this equation is called the total torsion of the plasma loop. The constant torsion of the solar loop allows us to compute

$$\int \tau_{0} ds \geq 2\pi{\times}10^{5} km \tag{III.15}$$

which is well within the observational limit based on the length of a flare loop, which is [8] $100000km$. Finally let us consider the computation of the ratio between magnetic energy E_M and the torsion energy E_T which yields

$$\frac{E_{M}}{E_{T}} = \frac{\frac{1}{8\pi}\int B^{2} dV}{\int \tau^{2} dV} \tag{III.16}$$

which yields

$$\frac{E_{M}}{E_{T}} = \frac{1}{8\pi} {B_{s}}^{2} {\alpha^{2}}_{twist} {r^{2}}_{0} \approx 10^{9} \tag{III.17}$$

IV Conclusions

An important issue in plasma astrophysics is to have analytical models for solar loops based upon astronomical and solar physics data. In this paper we discuss the several features of the Riemannian geometrical model of the plasma loop in the solar corona, to test it against these data. We show that all tests made are very well within the data of the EUV solar loops. Dynamo action investigations with a nonsteady dynamo model could be ineteresting to be tested against to these data. These computations may appear elsewhere.

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